Research Progress Report
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The project I worked on this past summer addresses the challenge of characterizing and predicting the collective energy landscape of quantum systems involving interactions between many electrons. The challenge in modeling these kinds of systems is that the size of the quantum system grows exponentially with the number of particles. For example, if you consider only spin for a system of 100 electrons, the size of the relevant Hilbert space is 2^{100}. When you consider additional parameters such as orbital states for each electron, the space is even larger. For systems with many particles, these kinds of exact quantum-mechanical calculations become computationally intractable.

To address this challenge, we use methods from network science to model quantum antidots, which are an example of quantum systems involving interactions between many electrons. In a quantum antidot, electrons occupy specific energy levels in a similar way that electrons in atoms occupy specific molecular orbitals. We consider an energy state of a quantum antidot to be the specific way in which the energy levels are occupied by electrons, and we consider transitions between energy states as the change in the way in which these specific energy levels are occupied after the tunneling event of a single electron into or out of the quantum antidot. Instead of studying interactions between individual electrons, we create a network model that considers how the energy state of the entire system changes as additional electrons tunnel in and out of the quantum antidot.

A network consists of two types of components: nodes and edges. Nodes represent discrete entities (such as people, web pages, or subway stops), and edges represent relationships or connections between these nodes (such as friendship, hyperlinks, or subway tracks). For our network representation of quantum antidots, the nodes are energy states and the edges are transitions between energy states. Network science provides a mathematical framework to characterize and describe complex patterns in systems that can be represented as networks.

We constructed network representations for the quantum antidot over a variety of voltage settings and temperatures. From the voltage settings and temperature, we can calculate the physical quantities of current and differential conductance through the quantum antidot. We found that calculating a specific measure of network complexity known as Rent's topological exponent is sufficient to distinguish between frustrated and non-frustrated voltage regimes. Furthermore, we found that this measure of complexity is a good predictor of current and differential conductance in non-frustrated regimes. To the best of my knowledge, this is the first study that connects networks-statistical measures with the empirical physical quantities of a quantum system.

With funding from the University Scholars program, I was able to present preliminary findings of this project as a contributed talk at the Complex Quantum Networks satellite of the 2018 flagship conference of the Network Science Society (NetSci 2018) in Paris. With support from the University Scholars program, I also prepared a manuscript detailing our methodology and findings for submission to a peer-reviewed journal.